A TENTATIVE APPROACH TO TEACHING AND LEARNING MATHEMATICS THROUGH METAPHOR

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Abstract
This paper looks at the role metaphor plays or might play in the teaching and learning of mathematics in school. Interpretations of what is meant by metaphor are considered and the ways in which metaphor is used within mathematics itself are explored. Drawing upon examples of metaphor used by teachers and students, this paper examines how metaphor can aid communication and understanding and draws attention to possible problems inherent in its use. Ideas discussed in this paper are likely to be of interest not only to those involved in mathematics education but to teacher educators in general.

1. What is metaphor?

An interest in metaphor tends generally to be regarded as the province of those concerned with the study of language and literature and one might be excused for suggesting that metaphor has little to do with the process of teaching and learning mathematics. It is to the area of literature that we turn first, to somewhat paradoxically perhaps, in order to begin to dispel the notion that metaphor is simply a piece of language or in Donald Schon’s words “window dressing for language” Schon 1967 12.

William Golding’s novel The Inheritors tells of a tribe of prehumans, some of whom are able to form pictures in their minds and can use likeness to help them make sense of their world.

Loki, one of the prehumans discovered “like” He had used likeness all his life without being aware of it. Fungi on trees were ears the word was the same but acquired a distinction by circumstances that could never apply to the sensitive things on the side of his head. Now in a convulsion of understanding Loki found himself using likeness as a tool as surely as he had ever used a stone to hack at sticks and meat. William Golding’s The Inheritors.

It is suggested that metaphor is not simply a piece of language for which there exists a literal equivalent. Metaphor is also a process of thought. Most importantly metaphor provides the possibility for talking about things we have never before talked about nor perhaps even thought of” Pollio et al 1977 11 Turning to Loki metaphor FUNGI ARE EARS the central components of a metaphor can be illustrated Metaphor is giving something a name that belongs to something else” Schon 1967 40 A metaphor then has two subjects a primary subject in this case FUNGI — the new concept and a secondary subject in this case EARS — a well established concept and it involves a comparison between the two subjects A final important feature of a metaphorical statement is that it is meaningless if taken literally — Fungi are not ears it is precisely this latter component which makes a metaphor so striking.

Our concern here is primarily with the metaphors which people produce to help them deal with new experiences whether describing something they never

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seen heard tasted or felt before or seeking to comprehend a new idea. It has been suggested that "we examine the unknown" scanning it over and over until we can describe it in terms of the known. Sutton 1978] 11] We intend to draw attention throughout this paper to what is described as the "interactive" role of metaphor Black 1979] 12] 83] which approaches metaphor functionally rather than grammatically Ortony et al. 1978] 923]

We want to draw attention now to another feature of metaphor using literature once again as the stimulus for our discussion

Language is the net that holds thought trapped within a particular culture
David Lodge Small World

Whorf 1972] 83] suggested that since different languages employ diverse metaphors to describe actions emotions and ideas one way of seeing the world will be influenced by those metaphors and hence will not necessarily be the same from culture to culture This idea has been echoed by various writers see for example McCloskey 1964] Schon 1967] but has been pursued most fully by Lakoff and Johnson 1980] in their book Metaphors We Live By It is their hypothesis that human thought processes are largely metaphorical so that the human conceptual system is metaphorically structured and defined. Hence "metaphors as linguistic expressions are possible precisely because there are metaphors in a person's conceptual system" Lakoff Johnson 1980] 6] or to express this more simply we speak metaphorically because we think metaphorically] Working from this idea Lakoff and Johnson have examined "frozen" metaphorical expressions] those which have become an accepted part of language in detail in order to study the nature of metaphorical concepts and also the way in which they influence our actions The "systematic" metaphors which they unearth structure the way in which the object being described is viewed accentuating some aspects while suppressing others a process described as "stressing and ignoring" Gattegno 1971] 11]

To clarify Lakoff and Johnson's use of the term "systematic" consider the metaphor TIME IS MONEY a dominant metaphor in both English and Chinese This metaphor structures our perceptions of time which can be "bought" "saved" "borrowed" etc This metaphor alone only partly structures the concept and other structural metaphors may also be employed such as TIME IS A MOVING OBJECT "time flies" "time passes" "time is moving on" and TIME IS A MEASURABLE OBJECT The latter metaphor is not mentioned by Lakoff and Johnson but it] too contributes to our perceptions of time "length" of time "how much" time is available These three metaphors help us grasp an abstract concept by means of other concepts which we understand in clearer terms

It may be interesting at this point to try to uncover systematic metaphors which structure the concept "mathematics"

II] Systematic metaphors of mathematics

The following are some of the systematic metaphors of which we have become aware by reading about mathematics] However] we do not claim that this list is exhaustive]

[1] MATHEMATICS IS A BUILDING
   [ ] algebraic "structure" "construction" of proofs foundations of mathematics] The three cornerstones of mathematics are algebra topology and analysis Mathematical logic is more like the mortar which holds the bricks together Stewart 1975] 229]

[2] MATHEMATICS IS A PLANT TREE
   [ ] "roots" of mathematics "branches" "offshoots" "growth"

[3] MATHEMATICS IS AN ART FORM
   [ ] "elegance" of a proof intrinsic beauty of mathematics

[4] MATHEMATICS IS A LANGUAGE
   [ ] "translate" into symbols "expressions" "vocabulary" "syntax" algebraic shorthand

[5] MATHEMATICS IS A FORMAL GAME
   [ ] rules

[6] MATHEMATICS IS A JIGSAW
   [ ] interlocking pieces

[7] MATHEMATICS IS A SET OF TOOLS
   [ ] used to solve problems

What one might ask is the point of identifying these systematic metaphors relating to mathematics Our suggestion is that each of them offers a particular way of looking at mathematics which reflects a
particular philosophy of mathematics. This has been detailed elsewhere (Nelder 1984) and is beyond the scope of this paper. However, what is important for teachers is that we should be aware that the expressions we use to talk about mathematics may elicit particular images of mathematics and that we should take care not to contribute to a one-sided view of the subject.

Although there may be many systematic metaphors relating to a particular concept, each contributing a somewhat different perspective, the picture may still be incomplete. Hence one finds individuals constructing their own idiosyncratic metaphors which highlight points which are significant for them.

The Russian mathematician R. Shafarevitch said of mathematics that “it resembles an orchestra performing a symphony composed by someone. A theme passes from one instrument to another and when one of the participants is bound to drop his part, it is taken up by another and performed with irreplicable precision.” (see Davis & Hersh 1981:52).

In an address to the Mathematical Association, John Mason chose a striking metaphor to depict the mathematician working on a point of contention sorting out a mistake or tackling a novel problem.

Core awarenesses are the grains of sand that mathematicians have worked on and which grew into the polished pearls that we expound as the current mathematical account or theory.

III A metaphor for the teaching-learning process. The conduit metaphor

The previous two examples lead well into a consideration of the language we use to talk about teaching and learning. What Michael Reddy has described as The Conduit Metaphor (Reddy 1979). It may be broken down into three parts.

IDEAS ARE OBJECTS
LINGUISTIC EXPRESSIONS ARE CONTAINERS
COMMUNICATION ISSENDING

This metaphor is an integral part of the way in which communication is viewed in our society. So much so that it is hard to talk about communication without using it. So why one might ask should we want to avoid using it? Largely because of the ideas about knowledge and learning which it entails. It implies that words and sentences have meanings independent of either the context or the speaker. It also depicts a passive role for learners who need only to listen to what they are told rather than to reconstruct meaning for themselves. Hence

Teachers are those people who take knowledge down from the shelves on which it is displayed and hand it out to students who presumably need only memory in order to receive it. (Gattegno 1971:20)

However, we are all familiar with the phrases “I don’t get it, Miss” or “I don’t get it, Sir.” A consequence of this type of interaction is that when a student fails to learn, the student may blame the teacher who failed to get the ideas across, while the teacher blames the student who failed to extract the meaning from the words. There is no sense of cooperation or of the necessity to negotiate meanings so that learners may recreate for themselves the ideas which words and symbols offer them the opportunity to construct.

From a personal perspective, thinking about the conduit metaphor has made me more aware of the limits it may impose upon people’s perceptions of the teaching-learning process. We are too often to notice ourselves using such expressions. They are no longer a taken-for-granted part of our language, but we have not succeeded in replacing them.

A further hazard arises from the conduit metaphor which we feel is rather special to mathematics although teachers of other subjects may wish to take issue with this. Mathematical concepts which we shall call MATHEMATICS 1, are contained in the written or spoken mathematical expressions. MATHEMATICS 2. Hence, by a process known as “semantic pathology,” MATHEMATICS 1 and MATHEMATICS 2 become identified and “Mathematics” comes to mean the symbolic expressions, diagrams, graphs etc. rather than the abstract concepts. This may lead to a tendency for school mathematics not to be about anything — just strings of symbols to be manipulated according to sets of rules.

Mathematics lessons in schools are very often not about anything. You collect like terms or learn the law of indices with no perception of why anybody needs to do such things. Written submission included in the Cockcroft Report (1982: paragraph 462).
We conclude this section with a metaphor produced by Richard Skemp in an article entitled “The Silent Music of Mathematics” Skemp 1983 which poignantly alerts us to the dangers of this type of teaching.

For most of us mathematics needs to be expressed in physical actions and human interactions before its symbols can evoke the silent patterns of mathematical ideas like musical notes simultaneous relationships like harmonies and expositions or proofs like melodies.

Skemp goes on to suggest that traditional methods of teaching mathematics have had the result that the majority have been turned it off in childhood For these the music of mathematics will be always altogether silent.

IV Students use of metaphor

We want now to move on to look at examples of students using metaphor in the mathematics classroom. We have chosen these examples because they illustrate some of the roles which metaphorical thought may play. The examples are numbered for ease of future reference and unless stated otherwise arise from classrooms which we once attended.

1] After a few moments hesitation a six year old describes an ellipse It is a sort of round oblong Murray 1980.]

2] A seventeen year old describes one graph as a U-shaped valley whereas another is a hump back bridge.

3] A twelve year old describes a cube as “like a square” in contrast to a cuboid which is a hump back bridge.

4] “A Wall like it’s a bit like double negatives in English too So I think A must be A”

5] A group of students had been invited to “invent” methods for adding subtracting multiplying and dividing complex numbers. A girl student suddenly noticed that complex numbers were “like surds” checked on how to divide by surds which she only dimly recalled applied the same method to complex numbers and found that it worked.

6] A thirteen year old defines parallel lines as “two lines that match and go together like a railway track Parallel lines are straight across and the lines never go downwards. These two lines are always the same length as the other one. They can be as far apart as you want them to be” Nicholas 1991

The first two examples illustrate a use of metaphor which is essentially comparative aiding communication by enabling the speaker to describe a new mathematical idea in terms of something more familiar whether from within or outside the field of mathematics. It is this particular use of metaphor which has helped mathematicallanguage to expand as words are introduced to fill gaps in the mathematical lexicon because of some perceived similarity between the mathematical meaning and everyday meaning this is an example of a process called “polysemy” by which language grows. Examples of this are “tree” diagrams “similar” shapes “rational” and “irrational” numbers “the” domain and “range” of function “the” neighborhood of a point “the” nodes of a network “amicable” numbers “common” factors etc. When a speaker has been familiar with such expressions for many years their metaphorical quality may go unnoticed. However for a teacher it is important to explore students’ understandings of such metaphorical terms in order to find out what meanings if any these evoke and to clarify ambiguity where this arises. A striking example of the confusion which may be engendered by a word which has an everyday as well as a mathematical meaning is quoted in Nicholas 1991. He describes a student not understanding the expression “What is common to because the student thought that “common” meant “too much lipstick”

Examples 4 and 5 above are of metaphors produced by students which entail more than simple comparison. In each case the comparison results in new insights relating to the primary subject of the metaphor. Hence the perceived similarity between “complementing a complement” in set theory and “double negatives” whether in mathematics or English leads to what is known about double negatives suggesting a possible rule for complements of complements. In the same way the perception of a similarity between complex numbers of the form a + bi and of the form a + b leads to the transfer of a method for dividing surds to an analogous one for dividing complex numbers.

On first glance example 3 appears to be a comparative metaphor. However on further
consideration it seems likely that the comparison
drawn here between two three-dimensional shapes and
their two-dimensional analogues may generate some
insights into the mathematically significant properties
of cubes and cuboids.

Example 6 is somewhat different from the other
elements presented here in that although it was
spontaneously produced by a student in response to
requests for a definition of the word “parallel”, it
shows signs of teacher influence. Railway lines are a
common exemplar used by teacher of the property
“parallel”. It is perhaps best described as an
“embodiment” of the property. However, what the
student appears to have done here is to identify
parallel lines with railway tracks, making the
metaphor PARALLEL LINES ARE RAILWAY
TRACKS, and hence restricting the meaning of
parallel in several ways notably by limiting the
number of lines to two and requiring that they should
always be the same length.

V. Teachers’ use of metaphor

If as was suggested earlier metaphor is a basic
feature of human communication teachers will be
using metaphor all the time in their everyday speech as
they talk with their students. We have already
discussed how the metaphors teachers unconsciously
use when talking about mathematics may influence
their students’ perceptions of mathematics, and we
want now to look at some metaphors teachers use
often in a deliberate way to help their students reach
an understanding of abstract ideas. We shall call these
pedagogical metaphors. We want to offer for
discussion two common pedagogical metaphors.

ALGEBRA IS SHORTHAND
AN EQUATION IS A BALANCE

What we want to think about is

[a] How might a teacher make use of the
metaphor

[b] How might the metaphor enhance
understanding

[c] What drawbacks if any might there be to
the use of this metaphor?

A thorough analysis of both metaphors would take
more time than is available here and so we shall
restrict ourselves to examining the example
ALGEBRA IS SHORTHAND the metaphor AN
EQUATION IS A BALANCE is examined in some

It is common for teachers to suggest that letters
are short forms of words and the identification a
apples | b | bananas etc in is often made especially
when introducing students to the idea of collecting like
terms in order to simplify an expression. Using this
approach the expression a a | b | b b could be
simplified to 2a | 3b by collecting together the apples
and then collecting together the bananas – a link with
real-life sorting activities. Thus 2a | 3b would mean
two apples and three bananas. While such
identification may prove helpful as a starting point
there are dangers inherent in its use. It obscures the
true mathematical meaning of 2a | 3b which is “two
times the number of apples plus three times the
number of bananas” where a and b represent the
numbers of apples and bananas respectively. In the
metaphorical identification of letters and objects the
concept of variable is not only missed but its
development may be hindered. The results of one
research program Hart 1981 indicate that many
students do not progress beyond the stage of “letter as
object” in their understanding of algebra.

It will be interesting to discover whether the
introduction of algebra via mathematical investigations
which has become more common in recent years will
result in improved understanding of algebra. As a
result of engaging in investigative activity students
produce mathematical generalizations in words which
they then turn into algebraic “shorthand”. This
approach although still depending on the same
underlying metaphor puts the onus on the student to
generate the algebra. For example a student
investigating the relationship between length of the
perimeter of a “snake” made from unit squares

An Example of a Square Snake

and the number of squares used to make the snake
wrote

“2 × no of squares plus 2 equals sic the
perimeter" which he then wrote "in shorthand" as 2s 2 p

The comparison between algebra and shorthand is limited to the notion of writing something in a shorter way and as such ALGEBRA IS SHORTHAND is a more limited pedagogical metaphor than AN EQUATION IS A BALANCE

VII Why metaphor is useful \ possible drawbacks in using metaphor

i) Metaphor is useful

The reasons why metaphor is useful in the teaching/learning process have been touched upon throughout this paper and have been examined in detail elsewhere[see for example Nolder 1991 112]. We shall summarize the reasons briefly here.

1) Metaphor makes it possible to talk about X at all.

2) Metaphor makes it possible to relate new concepts systematically to things already understood.

3) Metaphor extends thought.

4) Metaphor compels attention.

ii) Drawbacks in using metaphor

We need also to be aware of the drawbacks in using metaphor[one of which have been mentioned earlier]. The following points which are drawn from an earlier paper[see also Nolder 1991 112][13] summarize possible problems incurred by the use of metaphor.

1) Metaphors can blinker or distort perception.

2) Some elementary metaphors fail to apply in more advanced contexts.

3) One metaphor may not adequately structure a concept.

4) Students lack of familiarity with the secondary subject of a metaphor may render the metaphor useless as an aid to understanding.

Within this paper we have looked at a limited number of student metaphors[teacher metaphors and metaphors inherent in mathematics itself]. In so doing we have only touched the tip of the iceberg. Our purpose has been to draw attention to the existence of metaphor in the mathematics classroom so that metaphor might make a more effective contribution to students' mathematical experiences.

Note


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